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ORIGINAL ARTICLE

Does longevity cause growth? A theoretical critique

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Abstract This paper challenges conventional wisdom by arguing that greater longevity may have contributed less than previously thought for the significant accumulation of human capital during the transition from stagnation to growth. This is because when parents make choices over the quantity and quality of their offspring, greater longevity positively affects not only the returns to quality but also the returns to quantity. The theory suggests that in contrast to longevity, improvements in health are more likely to generate quantity quality tradeoff. Finally, it shows the importance of controlling for fertility when empirically examining the impact of children's health on their education.

Keywords Longevity · Fertility · Human capital · Growth

1 Introduction

The conventional wisdom suggests that prolonging the period in which individuals may receive returns on their investment spurs investment in human capital and causes growth. This conventional wisdom dates back at least to the seminal work of Yoram Ben-Porath (1967), and is consistent with the historical relationship among

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longevity, education, and per-capita output, which have been increasing simultaneously and monotonically since the middle of nineteenth century.[1]

Prior to the second half of the nineteenth century education was not widespread. In England, the average years of schooling of the cohort born between 1801 and 1805 was 2.3 years and rose to 5.2 years for the cohort born between 1852 and 1856 (Matthews, Feinstein, & Odling-Smee, 1982). Similar patterns are observed for other European countries and the US.[2] Furthermore, the high rates of child labor in Europe and the US during the nineteenth century, suggest that parents have had much control over the allocation of their children's time.[3] Hence, to understand the transition from stagnation to growth, the most reasonable framework would be one in which education choices are made by parents rather than by individuals themselves.[4] Moreover, parents do not choose the level of education of their children solely, but in combination with fertility choice. Indeed, the vast majority of the literature that emphasizes the role of human capital as the prime cause for the transition from stagnation to growth have assumed such a framework.[5]

Our paper shows that the Ben-Porath mechanism may fail to hold once parents make choices over education and fertility. The main contribution of the paper is to point out that greater longevity of children increases not only the returns to education but also the returns to fertility as each child lives longer. We show that if parental preferences are defined over the full income of their children, as in Galor and Weil (2000), an increase in children's longevity increases each child's income proportionally, irrespective of her level of education. Thus, it does not change the relative return between education (quality) and fertility (quantity) and, hence, does not cause any increase in the level of education chosen by the parents.[6] We call this the "neutrality result."

Our insight that gains in longevity increase also the returns to quantity, and thus mitigate the positive effect of longevity on education, can explain why recent studies fail to find positive effect of life expectancy on schooling outcomes. For example, Acemoglu and Johnson (2006) build an instrument for life expectancy using the pre-intervention distribution of mortality from various diseases around the world and the dates of global health interventions that began in the 1940s. They find a positive effect of life expectancy on fertility and no effect of life expectancy on schooling.[7]

[1] Numerous studies that explore the transition from stagnation to growth utilize this mechanism. See Ehrlich and Lui (1991), de la Croix and Licandro (1999), Kalemli-Ozcan, Ryder, and Weil (2000), Boucekkine, de la Croix, and Licandro (2002, 2003), Soares (2005), Cervellati and Sunde (2005), among others.

[2] See Flora, Kraus, and Pfenning (1983) for Europe and US Bureau of the Census (1975) for the US.

[3] See Basu (1999) and the references therein. All the empirical literature that investigate the phenomenon of child labor, either in the past of the nowadays developed economies or in contemporary developing economies, assumes that parents allocate the time of their children between child labor and schooling.

[4] This is not to say that individuals do not invest in their own human capital. However, we argue that the major part of investment in human capital at that time was done by parents.

[5] See Becker, Murphy, and Tamura (1990), Ehrlich and Lui (1991), Galor and Weil (2000), Galor and Moav (2002), Greenwood and Ananth (2002), Hazan and Berdugo (2002), Lucas (2002), Doepke (2004), Doepke and Zilibotti (2005), among others.

[6] Moav (2005) discusses this result without formalizing it.

[7] Lorentzen, McMillan and Wacziarg (2005) pursue a structural econometric approach to explore the effect of adult mortality on economic development. In contrast to Acemoglu and Johnson (2006), they

The discussion above weakens the argument that the rise in longevity have had a positive effect on the acquisition of human capital during the transition from stagnation to growth. However, the strong positive correlation among the two variables suggests that there might have been a third variable that has affected both education and longevity. One such variable may be health.[8] Health as a determinant of growth has been analyzed in two strands in the literature. The first strand assesses the direct effect of health on productivity. Seminal contributions are Fogel (1994) and Shastry and Weil (2003).[9] The second strand is closer to our argument as it assesses the indirect effect of health on income through education. Alderman, Behrman, Lavy, and Menon (2001), Bleakley (2007), Miguel and Kremer (2004) and Behrman and Rosenzweig (2004) estimate the impact of health on education. Most of this literature finds positive causal effect running from health to education. Closest to our argument comes the paper by Bleakley and Lange (2006) that finds that the eradication of hookworm disease in the American South circa 1910 led to an increase in school attendance and literacy rates, substantial gains in income and a reduction in fertility.

We incorporate health into the model by assuming that it joins education as an input in the production of human capital. We assume that the production function exhibits positive and decreasing marginal product in health and education and that the two inputs are complements. A naïve conclusion would be that the complementarity assumption is sufficient to assure that improvements in health would increase the investment in quality. Health improvements, however, not only increase the return on quality but also raise the level of human capital, i.e., the return on quantity. Consequently, the optimal level of education will rise only if the return on quality increases by more than the return on quantity. Intuitively, this would be the case if the degree of complementarity between health and education is sufficiently high.

Although the effect of health improvements on schooling is an empirical matter, our contribution here is two folds. First, we show the importance of controlling for fertility choice when empirically investigating this question. All of the aforementioned papers have ignored the endogeneity of fertility, with the notable exception of Bleakley and Lange (2006) who explicitly examine the effect of health improvement on education and fertility. Second, we show that if indeed health and education exhibit a sufficiently high degree of complementarity in the production of human capital, improvements in health can generate a transition from stagnation to growth that is consistent with the evidence.

Our paper shows that in the framework in which longevity is neutral, health can induce quantity-quality tradeoff. In this respect we argue that improvements

Footnote 7 continued
find that adult mortality positively affect fertility. Their result concerning the effect of adult mortality on investment in human capital is inconclusive.

8 Although longevity and health have been used by the empirical literature interchangeably, at the theoretical level they are differentiated because longevity measures the length of life while health measures one's physiological condition at a given point in time. In the context of this paper, longevity measures the length of productive life whereas health measures labor productivity per unit of time.

9 Fogel (1994) estimates the increase in energy available to the British population between 1790 and 1980 and argues that the increase in caloric intake boosted labor-force participation and the intensity of work per hour. Fogel traces roughly one-third of per-capita income growth in England during that period to this increase in labor input. Similarly, using current cross-country data, Shastry and Weil (2003) estimate the direct contribution of health to cross-country differences in per-capita output and find that health may account for one-third of the variation that is left unexplained by other measures of factor accumulation.

in health are more likely to generate quantity–quality tradeoff than gains in longevity. Interestingly, Bleakley (2006) finds a natural experiment that bridges between health and longevity. In Colombia, most of the malarious areas were afflicted with *vivax* malaria, a high-morbidity strain. However, significant portions of the country suffered from elevated rates of *falciparum*, a malaria parasite associated with high mortality. When he estimates an interacted model, he finds that eradicating *vivax* malaria produced substantial gains in human capital and income, while on the other hand, estimates indicate no such gains from eradicating *falciparum*.

The rest of the paper is organized as follows. Section 2 presents a two period model that formalizes our arguments with respect to the Ben-Porath mechanism under exogenous and endogenous fertility. Under exogenous fertility we show that the Ben-Porath (1967) mechanism is less robust than it seems and requires additional assumption on parental preferences. We then proceed to show that even when this assumption is met, the Ben-Porath (1967) mechanism fails to hold, once the framework is extended to allow for fertility choice. Section 2 ends with the incorporation of health into the production of human capital where we derive the formal condition under which improvements in health may enhance parental investment in the education of children. Section 3 presents an infinite horizon dynamic model in which improvements in health generate an evolution of an economy from stagnation to growth that is consistent with the historical evidence. Section 4 presents some concluding remarks.

2 The model

The model consists of two periods, t and $t+1$, and there is no discounting of the future by any agent. It is assumed that a representative adult possesses linear technology, making marginal productivity constant and is set equal to 1. At the beginning of period t, she decides how much to consume, c_t, how many children to have, n_t, and how much education to give each child, e_{t+1}. The adult lives a fraction π_t of period t and is endowed with h_t units of human capital. Thus, she divides her full income, between child raising and consumption.[10]

Let $\tau + e_{t+1}$ be the time cost for an adult of producing a child with educational level e_{t+1}. That is, τ is the time needed to raise a child irrespective of quality and e_{t+1} is the time devoted to each child's education. Hence, the time-cost of raising n_t children at educational level e_{t+1} is $(\tau + e_{t+1})n_t$. In period $t+1$, each child becomes an adult who lives a fraction π_{t+1} of the period.

Each level of education is translated into human capital according to the production function $h(e)$, where $h(\cdot)$ is assumed to be twice continuously differentiable, strictly increasing, and strictly concave.

Parental utility is denoted by $W_t = W(c_t, n_t\pi_{t+1}h(e_{t+1}))$, i.e., the parent's preferences are defined over household consumption as well as the full income of her offspring. Following Becker (1991), we assume that W_t is separable. Thus:

[10] Kalemli-Ozcan (2002) argues that when there is a precautionary demand for children, declining child mortality-another important aspect of increase in life expectancy-may have a strong negative effect on fertility and a positive effect on education. Doepke (2005) shows quantitatively that the incorporation of sequential fertility choice eliminates the impact of the decline in child mortality on fertility. We abstract from uncertainty in order to focus on the (deterministic) effect of longer productive lives of children on the decisions of their parents.

$$W_t = U(c_t) + V(n_t\pi_{t+1}h(e_{t+1})), \tag{1}$$

where U and V are both twice continuously differentiable, strictly increasing, and strictly concave.[11]

The adult in period t faces the following budget constraint:

$$\pi_t h_t = c_t + (\tau + e_{t+1})n_t h_t. \tag{2}$$

2.1 Longevity and exogenous fertility

In order to examine the mechanism proposed in Ben-Porath (1967) in a framework in which the parent chooses the level of education of her children, we assume in this section that fertility is exogenous. To simplify, we set $n_t = 1$. Maximizing (1) subject to (2) yields the following first-order condition:[12]

$$U'(c_t)h_t = V'(\pi_{t+1}h(e_{t+1}))\pi_{t+1}h'(e_{t+1}). \tag{3}$$

The left-hand side (henceforth: LHS) of (3) is the marginal cost of educating a child, measured in terms of the loss of utility from foregone consumption, and the right-hand side (henceforth: RHS) of (3) is the marginal utility of educating a child in terms of the utility gain from an increase in the child's full income. Note that the LHS of (3) is continuously increasing in e_{t+1} while the RHS of (3) is continuously decreasing in e_{t+1}. We assume the existence of an interior solution, denoted by e^*_{t+1}, that satisfies (3).

The LHS of (3) is independent of the longevity of the child, π_{t+1}, whereas the RHS of (3) may decrease, increase, or be independent of π_{t+1}. Note that the RHS of (3) is composed of two elements. The first element, $V'(\pi_{t+1}h(e_{t+1}))$, is the marginal utility that a parent derives from the child's full income. The second element, $\pi_{t+1}h'(e_{t+1})$, is the change in the child's full income for a marginal increase in education. Since the two elements act in opposite directions, the Ben-Porath mechanism is not robust to the assumption who chooses the optimal level of education. Intuitively, as the child lives longer and therefore her lifetime earnings increase, the marginal utility that the parent derives from her child's well being decreases, which, in turn, mitigates the positive link between longevity and education.

Therefore, an increase in the longevity of the child has a positive effect on education if and only if:

$$-V''(\pi_{t+1}h(e_{t+1}))\frac{(\pi_{t+1}h(e_{t+1}))}{V'((\pi_{t+1}h(e_{t+1}))} < 1.^{13} \tag{4}$$

[11] Note that nothing hinges on the separability of U and V.

[12] To highlight the role of longevity, we focus on an interior solution for e throughout Sect. 2.

[13] Note that the LHS of Inequality (4) is the elasticity of $V'(\cdot)$ with respect to $\pi_{t+1}h(\cdot)$. Therefore, Inequality (4) implies that the percentage change in $\pi_{t+1}h(\cdot)$ is greater than that of $V'(\cdot)$ for a marginal increase in education. For example, the CRRA utility function, $W_t = \frac{1}{1-\gamma}c_t^{1-\gamma} + \frac{1}{1-\gamma}(\pi_{t+1}h(e_{t+1}))^{1-\gamma}$ with $\gamma < 1$ satisfies Inequality (4).

This elicits the following proposition:

Proposition 1 *"The Modified Ben-Porath Mechanism." When fertility is exogenous, an increase in children's longevity increases the optimal educational level if and only if Inequality* (4) *holds.*

2.2 Longevity and endogenous fertility

In this part we examine whether gains in longevity can induce an increase in education in the framework in which fertility is endogenous. By treating education and fertility as a parental choice, we obtain the following first-order conditions:

$$U'(c_t)n_t h_t = V'(n_t \pi_{t+1} h(e_{t+1}))n_t \pi_{t+1} h'(e_{t+1}), \tag{5}$$

and

$$U'(c_t)(\tau + e_{t+1})h_t = V'(n_t \pi_{t+1} h(e_{t+1}))\pi_{t+1} h(e_{t+1}). \tag{6}$$

Note that (5) resembles (3) except that fertility is endogenous. The LHS of (6) is the marginal cost of quantity, measured in the utility loss from foregone consumption, and RHS of (6) is the marginal utility from quantity, measured in the utility gain from an increase in the children's full income. Solving (5) and (6) yields:

$$\frac{1}{\tau + e_{t+1}} = \frac{h'(e_{t+1})}{h(e_{t+1})}, \tag{7}$$

where the LHS of (7) is the relative price of education in terms of fertility and the RHS of (7) is the marginal rate of substitution between education and fertility.

Note that the marginal rate of substitution between education and fertility is independent of children's longevity because longevity has a symmetrical effect on the marginal utility from fertility and the marginal utility from education. This leads us to the following proposition:

Proposition 2 *"The Neutrality Result." When fertility is endogenous, an increase in children's longevity has no effect on the optimal level of education.*

Proposition 2 suggests that the positive effect of the prolongation of productive life on the acquisition of human capital obtained in growth models depends either on the assumption that fertility is exogenous when inequality (4) holds, or on non-homothetic preferences of parents.[14] Notice that even if parental preferences are non-homothetic, the neutrality result suggests that quantitatively, the effect of greater longevity is less than previously emphasized in the literature. This is because the literature has ignored the positive effect of longevity on the returns to quantity. Increases in longevity may also affect the wage profile over the life cycle. A well known fact from the labor literature is that labor earnings over the life cycle are hump-shaped. In contrast, for simplicity, our model assumes that wages are constant over the life cycle. Notice, however, that as long as wages increase proportionally over the life cycle for all levels of education, our analysis remains valid.[15]

[14] The result derived here relies on the homothetic preferences of parents with respect to the quantity and quality of their children. Specifically, we could rewrite the utility function as $W_t = U(c_t) + V(\pi_{t+1} n_t, \pi_{t+1} h(e_{t+1}))$. If (V_1/V_2) is independent of π_{t+1} the neutrality result follows.

[15] Formally, suppose one posits the following Mincer wage regression: $\ln w_i = \alpha_0 + \alpha_1 \text{schooling}_i + \alpha_2 \text{age}_i + \alpha_3 \text{age}_i^2 + \alpha_4 (\text{schooling}_i \cdot \text{age}_i) + \epsilon_i$. As long as $\alpha_4 = 0$, the neutrality result holds.

2.3 Health and endogenous fertility

In this part we examine whether improvements in health can account for the accumulation of human capital in the framework in which a rise in longevity is neutral. In view of the evidence surveyed in the Introduction, the most general way to incorporate health into our analysis is to assume that health is an input in the production of human capital. Formally, let the production function of human capital take the form:

$$h = h(e_{t+1}, \theta_{t+1}), \tag{8}$$

where θ_{t+1} is the level of health of each child. We assume that $h(e_{t+1}, \theta_{t+1})$ is an increasing, strictly concave function of both arguments with $\lim_{e\to\infty} h_e(e_{t+1}, \theta_{t+1}) = 0$ and $\lim_{\theta\to\infty} h_\theta(e_{t+1}, \theta_{t+1}) = 0$. Furthermore, we assume that education and health are complements in the production of human capital, i.e., $h_{e\theta}(e_{t+1}, \theta_{t+1}) > 0$.

By solving the maximization problem in Sect. 2.2 with the modified human-capital production function, we obtain:

$$\frac{1}{\tau + e_{t+1}} = \frac{h_e(e_{t+1}, \theta_{t+1})}{h(e_{t+1}, \theta_{t+1})}. \tag{9}$$

Note that for a given value for θ_{t+1}, (9) has a unique solution for e_{t+1}. The solution suggests another counterintuitive result. While one may think that the complementarity of health and education suffices to ensure that improvements in health will tip the coin in favor of quality at the expense of quantity, this is not necessarily so. Inspection of the RHS of (9) suggests that although improvement in health increases the marginal return on quality, it also increases the marginal return on quantity. Thus, the marginal rate of substitution between the two may increase, decrease, or remain unchanged. Thus, our theory shows the importance of controlling for fertility when empirically examining the impact of health on education.

Formally, health improvements will have a positive effect on education investment if and only if:

$$\frac{\partial [h_e(e_{t+1}, \theta_{t+1})/h(e_{t+1}, \theta_{t+1})]}{\partial \theta} > 0 \tag{10}$$

This leads us to the following proposition:

Proposition 3 *When fertility is endogenous, improvements in children's health have a positive effect on education investment if and only if Inequality* (10) *holds.*

Inequality (10) states that the percentage increase in h_{t+1} due to a marginal increase in e_{t+1} is increasing in θ_{t+1}. Technically, Condition (10) holds if the degree of complementarity between education and health is sufficiently high. It turns out that Inequality (10) holds for any constant return to scale (CRS) human capital production function in the range in which the elasticity of substitution between education and health is less than 1. The findings in Bleakley and Lange (2006) are consistent with a sufficiently high complementarity between education and health. As discussed in the introduction, they find that improvements in children's health induce parents to substitute education for fertility. Hereafter, we assume that (10) holds.

Thus far, our theory argues that longevity does not have a causal effect on education, despite the strong positive correlation between longevity and education among many European countries and Western Offshoots which has been observed since the second half of the nineteenth century. However, our theory proposes a way to reconcile this positive correlation among longevity and education with the proposed neutrality result. Assuming that better health promotes longer lives, our theory suggests that on the one hand, improvements in children's health promotes higher investment in education, while on the other hand, it induces greater longevity. In the next section, we portray the evolution of the economy from stagnation to growth that emphasizes the role of health and longevity in that process.

3 The evolution of the economy from stagnation to growth

This section demonstrates that the interaction between longevity, health, fertility and education, explored in the previous section, provides an alternative explanation for the transition from stagnation to growth. In this respect, the current paper is related to the strand of the literature that explains the long run transition from stagnation to growth.[16]

As evident from (9), it follows that as long as preferences are defined over household's consumption and the potential income of the children, as expressed in (1), the functional form of W_t does not affect the solution with respect to the quantity and the quality of the children. However, to facilitate the exposition of the dynamic system, we assume here that W_t takes the following form:

$$W_t = \alpha \ln(c_t) + (1-\alpha)\ln(n_t \pi_{t+1} h_{t+1}), \tag{11}$$

where $\alpha \in (0,1)$. Maximization of (11) subject to the budget constraint, (2), and the human capital production function, (8), yields the optimal consumption, c_t^*,

$$c_t^* = \alpha \pi_t h_t, \tag{12}$$

and the optimal number of children, n_t^*,

$$n_t^* = \frac{(1-\alpha)\pi_t}{\tau + e_{t+1}^*}. \tag{13}$$

From the first order condition with respect to the level of eduction, e_{t+1}, we define the function $G(e_{t+1}, \theta_{t+1})$ as follows:

$$G(e_{t+1}, \theta_{t+1}) \equiv h_e(e_{t+1}, \theta_{t+1}) - \frac{h(e_{t+1}, \theta_{t+1})}{\tau + e_{t+1}^*} \leq 0, \tag{14}$$

where $G(e_{t+1}, \theta_{t+1})$ is the difference between the benefit from a marginal increase in time invested in quality and a marginal increase in time invested in quantity. Thus, $e^* = 0$ if $G(0, \theta_{t+1}) < 0$ and $e_{t+1}^* \geq 0$ if $G(e_{t+1}^*, \theta_{t+1}) = 0$. To assure the existence of a corner solution in which the optimal time invested in education is zero, two additional assumptions are taken. First, $h(0,0) = 1$, i.e., when health is at its lowest level and parents do not invest in quality each individual has a positive level of human capital, which is normalized to 1. Second, $h_e(0,0) < \frac{h(0,0)}{\tau} = \frac{1}{\tau}$, i.e., when health is at its lowest level the benefit from an infinitesimal time invested in quality is smaller

[16] The economic development of Western Europe and the Western Offshoots over the long-run has been analyzed in the literature by Galor and Weil (2000), Jones (2001), Stokey (2001), Galor and Moav (2002), Lucas (2002), Hansen and Prescott (2002), Doepke (2004), among others. See Galor (2005) for a summary of these theories.

than an additional infinitesimal time invested in quantity. These two assumptions assure that $G(0,0) < 0$. Furthermore, it is assumed that $\lim_{\theta_{t+1}\to\infty} G(0,\theta_{t+1}) > 0$, i.e., when health approaches its highest level the benefit from a marginal increase in time invested in quality is higher than the benefits from a marginal increase in time invested in quantity for zero level of education. Given (10), $\forall(e_{t+1},\theta_{t+1}) \geq 0$, $G_\theta(e_{t+1},\theta_{t+1}) > 0$ and $G_e(e_{t+1},\theta_{t+1}) < 0$. Thus, noting that $G(0,\theta_{t+1})$ is continuous in θ_{t+1}, we have the following lemma:

Lemma 1 *There exists $\hat{\theta} > 0$ such that $G(0,\hat{\theta}) = 0$.*

Lemma 1 states that at $(0,\hat{\theta})$, the benefit from a marginal time invested in quality equals the benefit from a marginal increase in time invested in quantity. Further, using the implicit function theorem we have the following corollary:

Corollary 1 *There exists a single valued function, $e^*_{t+1} = e(\theta_{t+1})$, such that,*

$$e^*_{t+1} \begin{cases} = 0 \; if \; \theta_{t+1} \leq \hat{\theta} \\ > 0 \; if \; \theta_{t+1} > \hat{\theta} \end{cases} \tag{15}$$

where $e'(\theta_{t+1}) > 0, \quad \forall\theta_{t+1} > \hat{\theta}$.

As apparent from corollary 1, $e''(\theta_{t+1})$ depends on the third derivative of the production function of human capital. A concave relation between the level of education and the level of health is plausible in the relevant values of the model's parameters. We thus assume that:

$$e''(\theta_{t+1}) < 0, \quad \forall\theta_{t+1} > \hat{\theta}.^{17} \tag{A1}$$

From differentiation of (13) and (15) with respect to π_t and θ_{t+1} we have the following lemmas which will prove useful in the dynamics of the model:

Lemma 2 *An increase in longevity of the parents results in an increase in the parents' chosen number of children and has no effect on children's education.*

- $\frac{\partial n^*_t}{\partial \pi_t} > 0$,
- $\frac{\partial e^*_{t+1}}{\partial \pi_t} = 0$.

Lemma 3 *An increase in the health of the children results in a quality-quantity tradeoff: a decline in the parents' chosen number of children and an increase in the children's education.*

- $\frac{\partial n^*_t}{\partial \theta_{t+1}} \leq 0$,
- $\frac{\partial e^*_{t+1}}{\partial \theta_{t+1}} \geq 0$.

[17] This is a sufficient condition for the model's qualitative dynamics. A necessary condition is that $e''(\theta_{t+1}) < 0$ as θ_{t+1} approaches ∞. In any case, since the parent faces a time constraint, and in reality, integer constraint dictates that the optimal number of children cannot be less than 1, the time devoted to children's education is bounded from above. This justifies the concavity of $e(\theta_{t+1})$ as θ_{t+1} approaches ∞.

3.1 The determination of health and longevity

We assume that the level of health of a child in period t, θ_{t+1}, depends on the average income per-household in the economy, $\bar{y}_t$.[18]

$$\theta_{t+1} = \theta(\bar{y}_t), \tag{16}$$

where $\theta(y)$ is strictly increasing, strictly concave with $\theta(0) = 0$. Furthermore, we assume that $\theta(y)$ satisfies the Inada conditions: $\lim_{y\to 0}\theta'(y) \to \infty$ and $\lim_{y\to\infty}\theta'(y) = 0$.

We assume that the longevity of each child, π_{t+1}, is determined by the level of health of a child in period t, θ_{t+1}:

$$\pi_{t+1} = \pi(\theta_{t+1}), \tag{17}$$

where $\pi(\theta)$ is strictly increasing and strictly concave on $(0, \breve{\theta})$, $\pi(0) = 0$ and $\pi(\theta) = 1$ for all θ greater than some finite number denoted by $\breve{\theta}$.

3.2 The joint evolution of health, longevity, education and fertility

The level of education of a child in period t, e_{t+1}, is determined by her health at childhood, θ_{t+1}, as given by (15), whereas child's health, θ_{t+1}, is determined by the average level of income per-household in the economy, as given by (16). Finally children's longevity is determined by their health, θ_{t+1}, as given by (17). Inspection of Eqs. (15), (16) and (17) and lemmas 2 and 3 highlights the different roles of health and longevity in the allocation of resources toward children. An increase in parental longevity increases the total resources devoted to raising children while an improvement in children's health increases the relative returns to child quality. Substituting (8), (12), (17) and $\bar{y}_t = c_t$ into (16) implies that the level of health of each child is determined by the level of health of the parent as well as by the level of education of the parent, which according to (15) is determined by her health. Hence, we can express the health of each child, θ_{t+1} as a function of the health of her parent, θ_t:

$$\theta_{t+1} = \theta\left[\alpha\pi(\theta_t)h\left(e(\theta_t), \theta_t\right)\right] \equiv \psi(\theta_t), \tag{18}$$

where $\psi(\cdot)$ is a first order nonlinear dynamic system.[19] Hence (18) suggests that the sequence $\{\theta_t\}_{t=0}^{\infty}$ governs the evolution of the economy. We now turn to analyze the properties of this system.

Lemma 4 *The dynamic system $\theta_{t+1} = \psi(\theta_t)$ has the following properties:*

1. $\psi(0) = 0$
2. $\psi'(\theta_t) > 0 \quad \forall\theta_t \geq 0$
3. $\lim_{\theta\to 0}\psi'(\theta_t) > 1$

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[18] Notice that the model assumes homogeneity within a generation and hence household behavior describes the average behavior in society: c_t, y_t and n_t also equal the average consumption, income and fertility in society, respectively. Notice also that if the investment in children is done at home then $y_t = c_t$, while if it is done in the market then $y_t = \frac{c_t}{\alpha}$. In both cases, c_t proxies per-capita income in society since the log-linear utility function implies that c_t grows faster than population. Henceforth, we adopt the former interpretation which implies that $\bar{y}_t = c_t$.

[19] Notice that the dynamic system can also be written as $\left\{\theta_{t+1} = \phi(e_t, \theta_t), e_{t+1} = e(\phi(e_t, \theta_t))\right\}$ and analyzed it in the plane (e, θ). Our approach above is however, simpler as it reduces the dynamic system to one dimension.

4. $\psi'(\theta_t)$ *is discontinuous at* $\theta_t = \hat{\theta}$
5. $\lim_{\theta\to\infty} \psi'(\theta_t) = 0$.

Proof

1. Since $\pi(0) = 0$ and $\theta(0) = 0$, it follows that $\psi(0) = 0$.
2. Since $\pi'(\theta) \geq 0$, $h_\theta(e,\theta) > 0$, $h_e(e,\theta) > 0$, $e'(\theta) \geq 0$ and $\theta'(c) > 0$, it follows that $\psi'(\theta_t) > 0$.
3. The assumption $\lim_{c\to 0} \theta'(c) \to \infty$ and $\lim_{\theta\to 0} \alpha\pi'(\theta)h > 0$ assures that $\lim_{\theta\to 0} \psi'(\theta_t) > 1$.
4. Since $e'(\theta_{t+1}) > 0, \forall\theta_{t+1} > \hat{\theta}$ whereas $e'(\theta_{t+1}) = 0, \forall\theta_{t+1} < \hat{\theta}$, the derivative $\psi'(\theta_t)$ is discontinuous at $\theta_t = \hat{\theta}$.
5. Since $\pi(\breve{\theta}) = 1$ and $\pi'(\theta) = 0$ $\forall\theta > \breve{\theta}$, $\lim_{e\to\infty} h_e(e_{t+1},\theta_{t+1}) \to 0$ and $\lim_{\theta\to\infty} h_\theta(e_{t+1},\theta_{t+1}) \to 0$ assures that $\lim_{\theta\to\infty} \psi'(\theta_t) = 0$. □

Lemma 4 assures that the graph of $\psi(\theta_t)$ in the plane (θ_t, θ_{t+1}) goes through the origin, its slope at the origin is greater than 1 and it has to cross the 45 degree line at least once at some finite θ_t. Note that in principle, $\psi(\theta_t)$ may cross the 45 degree line either once or three times. Furthermore, even if it crosses the 45 degree line only once, this may occur at $\theta < \hat{\theta}$ or at $\theta > \hat{\theta}$. The following assumption assures that $\psi(\theta_t)$ has a unique non trivial steady state which is larger than $\hat{\theta}$:

$$\lim_{\theta_t\to\hat{\theta}^-} \left\{\theta'\left(c(\theta_t)\right) \cdot \left[\pi'(\theta_t)h(0,\theta_t) + \pi(\theta_t)h_\theta(0,\theta_t)\right]\right\} \geq \frac{1}{\alpha}. \tag{A2}$$

Proposition 4 *Under* (A2)*, the dynamic system* $\theta_{t+1} = \psi(\theta_t)$ *has a unique stable steady state denoted by* $\bar{\theta}$.

Proof Follows immediately from Lemma 3 and (A2). □

Figure 1 presents the dynamic system $\theta_{t+1} = \psi(\theta_t)$.[20]

3.3 The process of development

As can be seen from Figure 1, the evolution of the economy is characterized by two distinct regimes corresponding to the observed dynamics of output per capita, population growth, health and education.[21] The first regime is characterized by low levels of health and education, an increase in population growth and minuscule growth rates of output per capita in modern standards. The second regime is characterized by secular increase in health and education, a decline in the growth rate of population and a rapid growth in output per capita. Galor and Weil (2000) refer to the former regime as the "Post Malthusian Regime" and the latter as the "Modern Growth Regime." The dynamics portrayed by the model accounts for the two regimes as well as the endogenous transition from the Post Malthusian to the Modern Growth Regime.

Consider an economy in early stages of development where the initial condition θ_0 is historically given and satisfying $0 < \theta_0 < \hat{\theta}$. In these stages the level of health and thus longevity increase monotonically which in turn increase the resources devoted

[20] Assumption (A2) assures that the slope of $\psi(\theta_t)$ as θ_t approaches $\hat{\theta}$ from below is at least 1. This assumption makes the consideration whether $\psi(\theta_t)$ is concave or convex for low values of θ_t redundant. Figure 1 presents one possible shape of $\psi(\theta_t)$.

[21] Galor (2005) provides a comprehensive survey of the evidence.

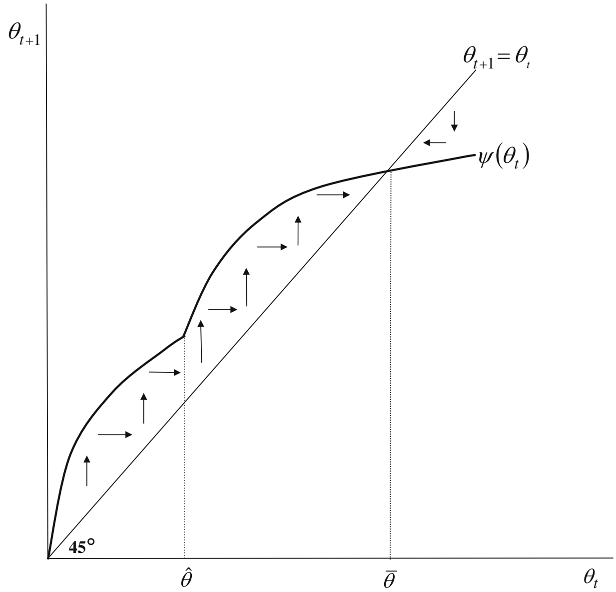

Fig. 1 The dynamic system

to raising children. Since health is still below the threshold level $\hat{\theta}$, education is zero and all the increase in resources is allocated to increase in fertility. At some period $\hat{t}$, health reaches the threshold level $\hat{\theta}$ and education starts to increase monotonically. Notice that from continuity there exists a time interval $[\hat{t}, \tilde{t}]$, such that as long as $t \in [\hat{t}, \tilde{t}]$, the optimal increase in education generated by the higher levels of health does not absorb the increase in total resources devoted to raising children, implying a concurrent increase in education and population growth.

The economy enters the Modern Growth Regime when $t = \tilde{t}$. In this regime, health is sufficiently high which implies that returns to education become sufficiently high to induce an increase in investment in education higher than the increase in resources allocated to child rearing. Thus, health, longevity and education increase while population growth starts its long run decline.

4 Concluding remarks

Contrary to conventional wisdom, we argue that greater longevity may have contributed less than previously thought for the significant accumulation of human capital during the transition from stagnation to growth. Our argument in based on the insight that once parents choose education in combination with fertility, greater longevity of the children, positively affects not only the returns to quality but also the returns to quantity. This mitigating effect can help explain recent empirical failures to find support for a positive effect of life expectancy on schooling, both in cross-country studies (Acemoglu & Johnson, 2006) and within countries (Bleakley, 2006).

Within the framework in which longevity is neutral, we argue that improvement in health can induce quantity–quality tradeoff. This theoretical argument finds strong support in Bleakley (2006) and has two implications. First, it suggests a new guideline for the empirical investigation of the relationship between health and education. In particular, our theory suggests that abstracting from fertility choice hides the true impact of health improvements on investment in education. Second, it proposes a way to reconcile the positive correlation among longevity and education by suggesting that on the one hand, improvements in children's health promotes higher investment in education, while on the other hand, it induces greater longevity.

Finally, we demonstrate how an economy can evolve from stagnation to growth by emphasizing the role of health and longevity in that process. Specifically, we show that on the one hand, gains in longevity ease parental budget constraint, inducing parents to invest more resources in their children, while on the other hand, health improvement steers these resources from quantity to quality. The model can then replicate three distinct phases that are consistent with the evidence. In the first phase, children's health is sufficiently low, and all additional resources devoted to children are channeled towards higher population growth. In the second phase, children's health status improved but it is not sufficiently high to justify the allocation of most of the additional resources devoted to children to education, and hence the initiation of formal education is accompanied by an increase in population growth. Finally, in the third phase, as health becomes sufficiently high, education absorbs an ever increasing portion of parental investment in children and hence population growth decelerates.

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